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DESIGN METHODOLOGY FOR ROUND WIRE COMPRESSION SPRINGS

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A concise, logical method for designing round wire, compression springs is provided. An overview of the method is that the type of available material (M) should first be selected on the basis of what type of service is required, i.e., fatigue life, operating temperature, large loads, cost. Then either performance (P) or space (S), not both, should be selected and the remaining parameter, S or P, will be determined via the standard spring equations. Essentially, algebraic equations can describe the approach, $P + M = S$ where P and M are selected and S is calculated, or $S + M = P$ where S and M are selected and P is calculated. An approach that is NOT recommended is to select P and S and calculate M ($P + S = M$). It is easily shown that this approach can result in the condition where the material required to provide the specified performance in the allocated space does NOT exist.

The background information (material available, manufacturing and testing considerations, fatigue, presetting) needed for designing a compression spring is provided. Also, guidance is provided on which parameters should be specified and how to determine the tolerances necessary for producibility and function.

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NOMENCLATURE

Symbol	Term	Unit
D_o	Outside diameter	inch
D	Mean coil diameter	inch
D_i	Inside diameter	inch
d	Wire diameter	inch
C	Spring index	inch/inch
F_s	Load at solid height	lb
L_f	Free length	inch
L_s	Solid height	inch
k	Spring rate ($\Delta F/\Delta L$)	lb/inch
N	Total number of coils	--
n	Number of active coils	--
τ_y	Shear yield strength	Ksi
τ_s	Shear stress at solid height	Ksi
K_{w1}	Correction factor for curvature and direct shear	--
K_{w2}	Correction factor for direct shear	--
S_t	Ultimate tensile strength	Ksi
S_y	Tension yield strength	Ksi
H	End fixation factor for end condition	--

INTRODUCTION

A review of several sources addressing the subject of compression springs has revealed that a concise, logical methodology for designing a spring is not readily available. In addition, information is often conflicting and has to be gathered from several sources in order to have a complete "set of tools" needed for designing a spring.

The purposes of this report are twofold:

- To present a concise, logical methodology for designing round wire, compression springs.
- To compile and discuss the important backup information needed for designing a spring.

METHODOLOGY

Three characteristics are associated with a compression spring:

- Performance (P): Performance is the ability to develop a load at a specific compressed length without resulting in dimensional change, premature fatigue failure, and possibly without buckling occurring.
- Material (M): Specific types of material are available with limitations on and ranges of strength, size, operating temperature, and fatigue life.
- Space (S): The size or dimensions of a spring will be either an output of or an input to the calculations. Often a spring has to be "squeezed" into a limited amount of space that remains after a design is completed and this may result in "problem" performance.

The design philosophy presented in this report is based on the cliche, "one cannot get something for nothing." Relating this to the design of a spring results in the understanding that to obtain the desired performance (P) with available material (M), requires adequate space (S). This can simply be expressed by the algebraic equation, $P + M = S$. Likewise, if space and material are selected, the performance is a consequence of $S + M = P$, hence the performance is calculated from the "spring equations." One could also conclude that $P + S = M$, however, it can easily be shown that the material required for specified performance and space does NOT always physically exist. It is recommended that a spring designer select either P or S but not both, select the M based on cost, fatigue, operating temperature considerations, and then calculate the space or performance that results from the "spring equations." It is felt that, too often, material is the "characteristic" that is "calculated" with the potential for catastrophic results.

BACKGROUND INFORMATION

The following items are considered to be important as background information:

- Material: Only plain carbon and alloy steel materials are considered. Table I lists specifications, strength, condition, diameter range, fatigue performance, control of decarburization, and maximum service temperature. Note the high carbon and alloy steel valve quality wires available (control of decarburization and surface defects) provide excellent fatigue performance. Preferred wire diameters (ref 1) are listed in Table II.

- Manufacturing/Testing Considerations:

1. Spring index (C) (ref 1): The preferred index ($C = D/_d$) range is 4 to 12. Springs with an index above 12 may tangle, hence may require individual packaging and springs with indexes lower than 4 are difficult to form.

2. Linearity of load (F) versus deflection (δ): The relationship of F versus δ (Hooke's Law) is a straight line ($F = k \cdot \delta$), where k is the slope of the line (spring rate). The linear relationship only exists for about the central 70 percent of the total deflection range, thus it is recommended that loads be specified at compressed lengths within the linear range; see Figure 1. During the initial 15 percent of deflection, the ends do not remain closed, hence the rate is less than theory and for the last 15 percent, some coils touch before solid length, hence the rate is greater than theory.

3. Buckling (ref. 2): Buckling of a compression spring is determined by the relationship of

$$\frac{HD}{L_f} \ to \ \frac{L_f - L_s}{L_f}$$

which is plotted in Figure 2. The value of H is a function of the end condition; see Figure 3. The plot of deflection/free length versus HD/L_f, shown in Figure 2, distinguishes between a stable versus unstable condition. A spring that buckles during its deflection should be contained in a cylinder or operate over a rod. Since the friction that results from the spring contacting the cylinder or rod will result in different loads than predicted by theory, larger load tolerances would be necessary. Guidance on testing of a spring that buckles has not been discovered. It is recommended that the loads/compressed lengths specified for performance requirements be selected in the deflection range that exists prior to buckling.

4. Types of ends (ref 1): Available types of ends are plain, plain and ground, squared, squared and ground. The dimensional characteristics for each type of end are as follows:

Types of Ends

Dimensional Characteristics	Plain	Plain and Ground	Closed or Square	Closed or Square and Ground
Solid Height (L_s)	$(N+1)d$	Nd	$(N+1)d$	Nd
Pitch (p)	$\frac{L_f - d}{n}$	$\frac{L_f}{n}$	$\frac{L_f - 3d}{n}$	$\frac{L_f - 2d}{n}$
Active Coils (n)	$\frac{L_f - d}{p}$	$\frac{L_f}{p} - 1$	$\frac{L_f - 3d}{p}$	$\frac{L_f - 2d}{p}$
Total Coils (N)	n	$n+1$	$n+2$	$n+2$
Free Length (L_f)	$pN + d$	pN	$pn + 3d$	$pn + 2d$

To improve squareness and reduce buckling during operation at least a 270° bearing surface is required (Grade A per MIL-S-13572). Squared ends (not ground) are preferred because of cost if the wire diameter is less than 0.020 inch or the spring index is large (C greater than 12).

- Fatigue (ref 1): Springs subjected to cyclic loading (greater than 1000 cycles) are vulnerable to fatigue failure. Fatigue life of a spring is dependent upon:

  1. The type of material selected and surface conditions allowed (decarburization and/or surface defects); see Table I.

  2. The applied stress (stress range and mean stress); see Table III.

  3. The residual stress on the outer portion of the wire generated by shot peening; see Table III.

The typical fatigue data includes τ_{max} versus N, where the stress ratio $R(\tau_{min}/\tau_{max})$ is zero; see Table III. The procedure for predicting the fatigue life for $R \neq 0$ is to plot the stress (τ_{max}) versus fatigue life data (N cycles) shown in Table III, on the Goodman diagram (45° line of τ_{max} on ordinate - τ_{min} on abscissa); see Figure 4. The number of desired cycles (N) is selected (point A),

a vertical line is drawn from A to the τ_{max} versus N curve (point B), a horizontal line is drawn from B to intersect the ordinate τ_{max} when R = O (point C), a line is drawn from C to the torsional strength (estimated at two-thirds the tensile strength) that is represented on the 45° line as point D. The line from C to D represents the stress combinations when R $\neq$ O that meet the desired life (see Example 1).

Shot peening provides two conditions to improve the fatigue lives of springs. First, shot peening creates a biaxial compressive residual stress on the surface that lowers the mean tensile stress, thereby resulting in a significant improvement in high-cycle (greater than 5×10^5 cycles) fatigue life; see Table III. Secondly, shot peening minimizes the effect of stress concentrations (pits, scratches, etc.), which are dramatically detrimental to springs made from high-strength (low ductility) material that is operated in the high-cycle régime.

If the spring is operated in the low-cycle regime, which means the applied stress range is high, the residual stress from shot peening becomes insignificant; i.e., it is overwhelmed and the stress magnification at stress concentrations becomes so large that a reduction in stress from shot peening is not significant. Low-strength ductile material becomes a better choice than high-strength, low ductility for springs operated in the low-cycle regime because stress concentrations are less detrimental (crack initiation does not occur as rapidly due to reduced notch sensitivity) due to the localized plasticity that can be created without cracking.

- Presetting: Presetting a spring imparts beneficial residual stress that increases the load-carrying ability and energy storage per pound of material. The spring's initial free length exceeds the desired final free length (at least 10 percent longer) and the applied stress exceeds the material's yield strength during the first application of load (spring is compressed to its solid height). This results in the plastic deformation (permanent set) necessary for creating favorable residual stress. After the presetting operation, minimal change in free length should occur and the load-carrying ability and energy storage per pound of material should be dramatically improved. Presetting increases the maximum shear stress allowable at solid height by approximately one-third or 33 percent and eliminates the correction factor for curvature; see Appendix I, Part IIB. The residual stress due to presetting has only a small effect on fatigue performance, and the estimated fatigue life values in Table III have taken into account the preset residual stress.

CALCULATIONS

The examples used for calculating either SPACE or PERFORMANCE or MATERIAL are included in Appendix II.

<u>Example 1: P + M = S (Buckling Not Allowed)</u>

Calculate the space (spring dimensions) with performance and material preselected. The performance selections include the initial load (F_1) or maximum load (F_2), the deflection between loads F_1 and F_2, and the minimum fatigue life required with shot peening. Buckling during the

total deflection ($L_f - L_s$) is not allowed. The material selection is ASTM A231, which is a readily available, reasonably priced, alloy steel wire. Selecting one load and the deflection between two loads allows calculations to result in a spring that will meet all requirements relative to dimensional stability (not overstressed), fatigue life, and buckling. If, for instance, one load and the length at that load were selected, the spring rate would also have to be selected which might result in a "dead end," i.e., a spring could not be selected that meets all requirements. The space calculations are only possible after deciding what portion of the total linear range that the spring will operate. Example 1 uses 15 to 85 percent, which is the maximum recommended. This choice results in specific values for the total deflection ($L_f - L_s$), the deflections during operation ($L_f - L_1$) and ($L_f - L_2$), the spring rate (k), and the second load (F_2). The next step is to determine the maximum shear stress allowed to provide the required fatigue life from the Goodman diagram, stress versus life plot shown in Figure 4. This allows the relationship between wire diameter (d) and spring index value (C) to be established from the shear stress equation, i.e., for a specific value of d, the maximum value of C is calculated. For the selected values of d (select a preferred value from Table II) and the calculated value of C for controlling the stress values, the number of active coils (n) is calculated to obtain the desired fatigue life. This allows caclulating L_s and L_f, which are necessary to determine if the spring is stable shown in Figure 3.

The balance between fatigue life and stability is interesting and worth noting. For a specific wire diameter (d), the stress levels increase as the mean coil diameter increases hence fatigue life <u>decreases</u> however the spring becomes <u>more stable</u>. The first wire diameter chosen (d of 0.406) resulted in a small mean coil diameter (D = 3.106 and C = 7.65), thus an <u>unstable</u> spring. Increasing the wire diameter to 0.437 requires a larger value of D (3.985) to guarantee that the fatigue life will result in a <u>stable</u> spring. It should also be noted that reducing the percentage of the total linear range that the spring operates (for example 70 percent versus 60 percent) will increase the stress ratio (τ_1/τ_2) and allow an increase in the stress levels for a particular fatigue life. This allows C or D to increase for each selected value of d which improves stability. Finally, the shear stress at solid height is calculated to determine if the spring has to be preset.

<u>Example 2: P + M = S (Buckling Allowed and Fatigue Not a Concern)</u>

The performance and material selected are the same as Example 1; however, the relationship between d and D is based on selecting a solid stress value that will prevent plastic deformation. The results of Example 2 show that reducing the stress levels reduces the mean coil diameter yet the number of coils and free length increase in order to maintain the required spring rate. It should be noted that although the coil diameter was reduced, the spring remained in the stable category.

<u>Example 3: S + M = P (Buckling Not Allowed)</u>

Possible performance is calculated from a preselected amount of space (maximum bore size and maximum free length) and type of material. A decision concerning buckling is also

necessary before performance parameters are calculated. Buckling is not allowed in this example. The performance calculations determine loads and compressed lengths and fatigue life. The first step is to determine the relationship between d and D based on the knowledge that the maximum OD of the spring has to be less than the bore size to allow for expansion. The second step is to determine the maximum value of $L_f - L_s/L_f$ to maintain stability from Figure 3 for each selected available wire diameter. This allows calculation of L_s and n. The value of n is increased somewhat (≈0.5 coil), which increases L_s and therefore decreases the total deflection range to provide safety from buckling. The next step is to select the deflection range that the spring will operate (same problem as in Example 1), which then provides the values of L_1, L_2, F_1, F_2. Shear stress at solid height (τ_s) is calculated to determine if preset is required. The final step is to determine the fatigue life from Figure 4 for each value of d. The final choice of which value of d is selected is predicated on the desirable values of F_1 and F_2.

Example 4: S + M = P (Buckling is Allowed)

The space and material selected are the same as Example 3. Since buckling is allowed, the performance is based on the maximum load at solid height (F_s). F_s is calculated from the stress equation based on the maximum shear stress allowed (τ_{max} allowed is reduced to provide a safety factor). The value of d is selected from Table II and D is calculated from the OD limitation as shown in Example 3. The number of active coils (n) needed to provide the value of F_s is calculated based on the relationship of F_s to spring rate (k) and deflection (L_f-L_s). The remaining steps are to calculate the spring rate (k), select the deflection range, calculate F_1, L_1, F_2, L_2, calculate τ_1 and τ_2, and determine fatigue life from Figure 4.

Example 5: P + S = M (Buckling is Allowed)

The performance requirements are the same as Example 1, except buckling is allowed and space is limited to a 2-inch diameter bore and free length of 10 inches maximum. The first step is to select the operating deflection range to calculate L_s and k. The next step is to determine the maximum value of C such that the OD is less than the bore diameter and the spring rate is provided. This is accomplished by arranging the spring equation such that d/n is a function of C^3 and substituting $L_s/n+2$ for d so that n is a function of C^3. A trial and error process is required: select C, calculate n, d, D, and OD until the OD value (D+d) is very close to the desired value. The maximum C allowed provides the maximum value of d, which results in the lowest shear stress. This example reveals that if the space is limited, the value of d will be small and the stress will be so large that material that would prevent plastic deformation would not exist.

Example 6: P + S = M (Buckling Not Allowed; Allow Larger Space Than Example 5)

The performance requirements are the same as Example 1 with space increased to a 5-inch diameter bore and free length of 15 inches maximum. The steps to determine if material is available to accommodate the solid stress values are the same as Example 5. The larger space in this example results in much less stress and all available materials as potential candidates.

SPECIFICATION REQUIREMENTS AND TOLERANCES

Appendix III provides guidance on which parameters should be toleranced and which should be reference values. Examples of how to calculate the load tolerances are also given. The parameters that should be toleranced include wire diameter, coil or outer diameter, and the two loads at specific compressed lengths. The other requirements that should be specified are the spring specification, MIL-S-13572, Type I, Grade A or B; the material per the selected ASTM specification; the type of ends; presetting if required; shot peening if required; and a protective coating if required. Phosphating is not specified because it results in pitting (stress concentrations). The reference values should be L_f and N, which provide guidance but allow adjustment for manufacturing variations. Two examples for calculating load tolerances are outlined in Appendix III. Essentially, the load tolerance is controlled by the variation in the spring rate (k) that occurs due to the needed tolerances for d and OD and for the anticipated variation in L_f (Table 5-4 of Reference 1), which is the adjustment expected for variation in the pitch spacing between coils, the modulus of rigidity (G), and the machining of the ends.

The potential for a slightly different method to specify parameters also exists. That is, only one toleranced load would be required instead of the two loads as shown in Appendix III. The free length (L_f) would have to be a toleranced requirement in order to control the spring rate (k).

SUMMARY

- The proposed method is to select the type of material (M) based on type of expected service, then select either performance (P) or space (S), and calculate the remaining parameter, either (S) or (P). The algebraic equations that express this concept are $P + M = S$ or $S + M = P$.

- If the designer requires specific performance (P) requirements to be provided in a limited space (S), the material (M) required may not be available. In other words, the approach of $P + S = M$ is not recommended.

- The subtleties of manufacturing/testing a spring have to be considered for a proper design to be accomplished. The spring index C range of 4 to 12 is an important parameter. The linearity of load-deflection between 15 and 85 percent of the total deflection has to be considered and selected. Prevention of buckling is a major factor that has to be considered. Fatigue also has to be considered.

- The tolerance calculation for the specified load or loads is based on the variation in k that occurs from the tolerances required for d, OD, and L_f.

REFERENCES

1. *Design Handbook*, Associated Spring, Bristol, CT, 1981.

2. British Standard 1726 Part 1, "Coil Springs, Guide for the Design of Helical Compression Springs," Buckinghamshire MK146LE, England, 1987.

3. Seely and Smith, *Advanced Mechanics of Materials*, 2nd Edition, John Wiley & Sons, Inc., NY, 1963, pp. 81-84.

4. A.M. Wahl, *Mechanical Springs*, 2nd Edition, McGraw-Hill Book Co., Inc., NY, 1963, pp. 260-261.

5. R. Farrara and J.H. Underwood, "The Effect of Manganese Phosphate Coatings on Fatigue Crack Initiation," U.S. Army ARDEC Technical Report ARCCB-TR-90018, Benet Laboratories, Watervliet, NY June 1990.

Table I. Round Steel Wire

	High Carbon Steel Round Spring Wire			
Spec. (ASTM)	A227	A228	A229	A230
Typical Composition	AISI 1065	AISI 1085	AISI 1065 and 1066	AISI 1070
Condition	Cold Drawn	Cold Drawn	Quenched and Tempered	Quenched and Tempered
Tensile Strength Range (Ksi) (See Note 1)	Class 1 - 147/283 Class 2 - 171/324	230/439	Class 1 - 165/293 Class 2 - 191/324	210/245
Diameter Range (Inch)	0.020/0.625	0.004/0.250	0.020/0.625	0.062/0.250
Decarburization Control (See Note 4)	No	No	No	Yes
Inclusion Control	No	No	No	No
Fatigue Performance	Poor	Good	Poor	Excellent
Max. Service Temp (°F)	250	250	300	300

Table I. Round Steel Wire (Cont'd)

Alloy Steel Round Spring Wire					
Spec. (ASTM)	A231	A232	A401	A877	A878
Typical Composition	AISI 6150	AISI 6150	SAE 9254	SAE 9254	Unknown
Condition	Annealed and Cold Drawn or Quenched and Tempered	Annealed and Cold Drawn or Quenched and Tempered	Annealed and Cold Drawn or Quenched and Tempered	Annealed and Cold Drawn or Quenched and Tempered	Annealed and Cold Drawn or Quenched and Tempered
Tensile Strength Range (Ksi) (See Note 1)	190/300	190/300	235/300	245/305	205/290
Diameter Range (Inch)	0.020/0.50	0.020/0.50	0.032/0.438	0.020/0.375	0.020/0.375
Decarburization Control (See Note 4)	No	Yes	No	Yes	Yes
Inclusion Control	No	No	No	Yes	Yes
Fatigue Performance	Good	Excellent	Good	Excellent	Excellent
Max. Service Temp. (°F)	425	425	475	475	425

Table 1. Round Steel Wire (Cont'd)

ASTM Spec. Legend:

A227 - Steel wire, cold drawn for mechanical springs
A228 - Steel wire, music spring quality
A229 - Steel wire, oil tempered for mechanical springs
A230 - Steel wire, oil tempered carbon valve spring quality
A231 - Chromium/vanadium alloy steel spring wire
A232 - Chromium/vanadium alloy steel valve spring quality wire
A401 - Steel wire, chromium/silicon alloy
A877 - Steel wire, chromium/silicon alloy valve spring quality
A878 - Steel wire, modified chromium/vanadium valve spring quality

Notes:

1. The tensile strength range is for the quenched and tempered condition, which is obtained only after a complete heat treatment consisting of austenitizing, liquid (oil) quenching, and tempering (ref 1). The tensile strength for calculation purposes is estimated to be the minimum value specified for largest wire diameter in the applicable ASTM specification.

2. Heat treatment (austenitizing, quenching, tempering) after forming results in distortion. Fixturing will reduce distortion, however, this is costly and is avoided if possible. Typically, heat treatment after forming occurs with wire diameter greater than one-fourth inch (ref 1).

3. Low temperature, stress relief treatment (375° to 800°F) is applied after forming springs of heat-treated material to reduce residual stress and dimensionally stabilize. Residual stress remaining from forming will reduce the load-carrying capability of the spring (ref 1).

4. Valve quality: amount of decarburization is controlled; no complete decarburization and some (≈0.0015 depth) partial decarburization.

Table II. Preferred Diameters for Spring Steel Wire

Metric Sizes (mm)			English Sizes (Inch)	
First Preference	Second Preference	Third Preference	First Preference	Second Preference
0.10			0.004	
	0.11		0.005	
0.12			0.006	
	0.14		0.008	
0.16				0.009
	0.18		0.010	
0.20				0.011
	0.22		0.012	
0.25				0.013
	0.28		0.014	
0.30				0.015
	0.35		0.016	
0.40				0.017
	0.45		0.018	
0.50				0.019
	0.55		0.020	
0.60				0.021
	0.65		0.022	
	0.70		0.024	
0.80			0.026	
	0.90		0.028	
1.00			0.030	
	1.10			0.031

Table II. Preferred Diameters for Spring Steel Wire (cont'd)

1.20				0.033
		1.3	0.035	
	1.4		0.038	
1.6				0.040
	1.8		0.042	
2.0			0.045	
		2.1		0.047
	2.2		0.048	
		2.4	0.051	
2.5			0.055	
		2.6	0.059	
	2.8		0.063	
3.0			0.067	
		3.2	0.072	
	3.5		0.076	
		3.8	0.081	
4.0			0.085	
		4.2	0.092	
	4.5		0.098	
		4.8		0.102
5.0			0.105	
	5.5		0.112	
6.0				0.120
	6.5		0.125	
	7.0			0.130

Table II. Preferred Diameters for Spring Steel Wire (cont'd)

		7.5	0.135	
8.0				0.140
		8.5	0.148	
	9.0			0.156
		9.5	0.162	
10.0				0.170
	11.0		0.177	
12.0			0.192	
	13.0			0.200
14.0			0.207	
	15.0			0.218
			0.225	
16.0			0.250	
				0.262
			0.281	
				0.306
			0.312	
			0.343	
			0.362	
			0.375	
			0.406	
			0.437	
			0.469	
			0.500	

Table III. Maximum Shear Stress (Ksi) Versus Fatigue Cycles

	Percent of Tensile Strength			
Material	ASTM A228, A231, A401		ASTM A230, A232, A877, A878	
Fatigue Life (Cycles)	Not Shot Peened	Shot Peened	Not Shot Peened	Shot Peened
10^5	36	42	42	49
10^6	33	39	40	47
10^7	30	36	38	46

Note: Fatigue cycle data from Table 5-3 of Reference 1.

Stress ratio (R) = $\tau_{min}/\tau_{max} = 0$.

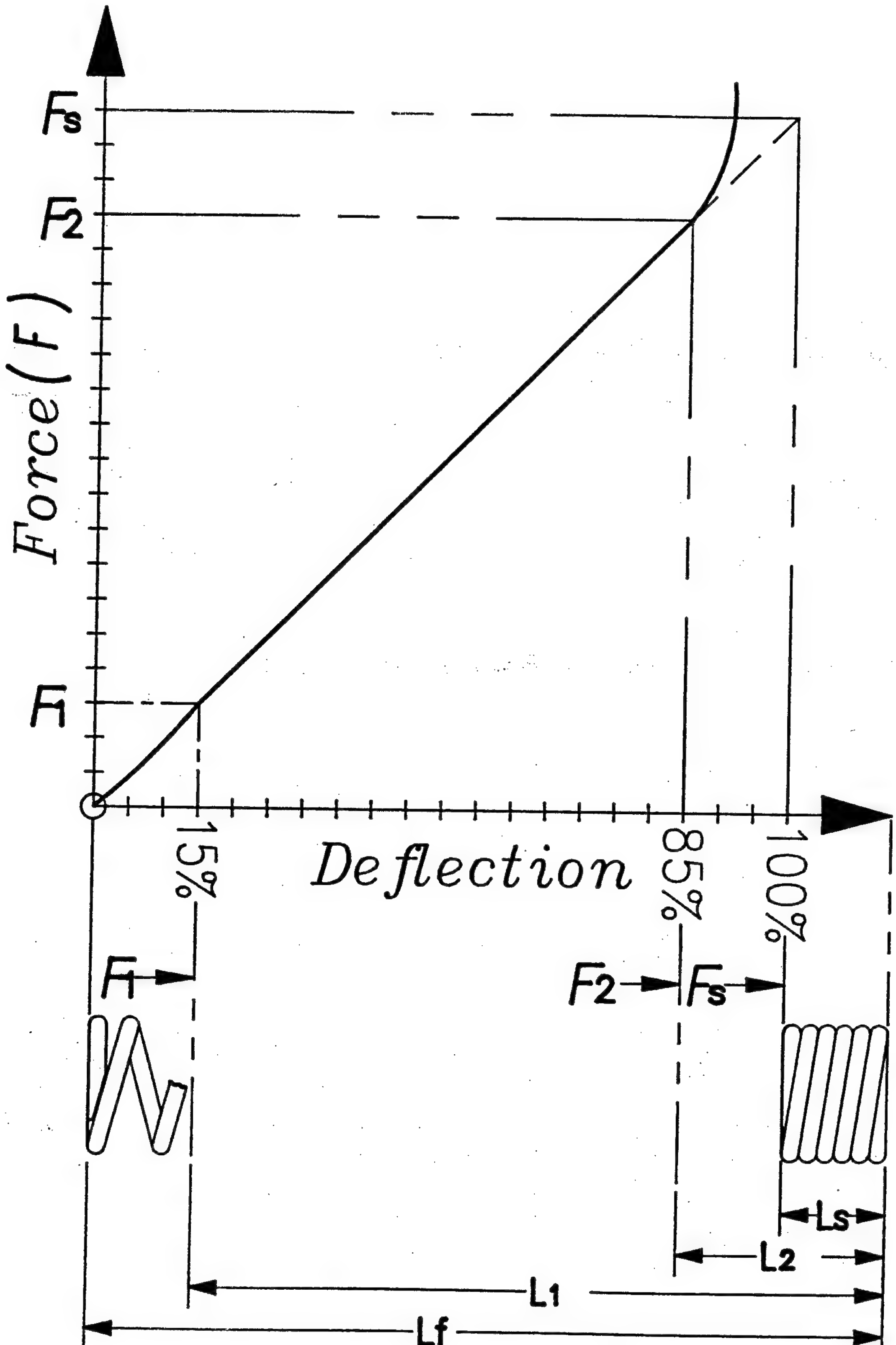


Figure 1. Typical force-deflection curve.

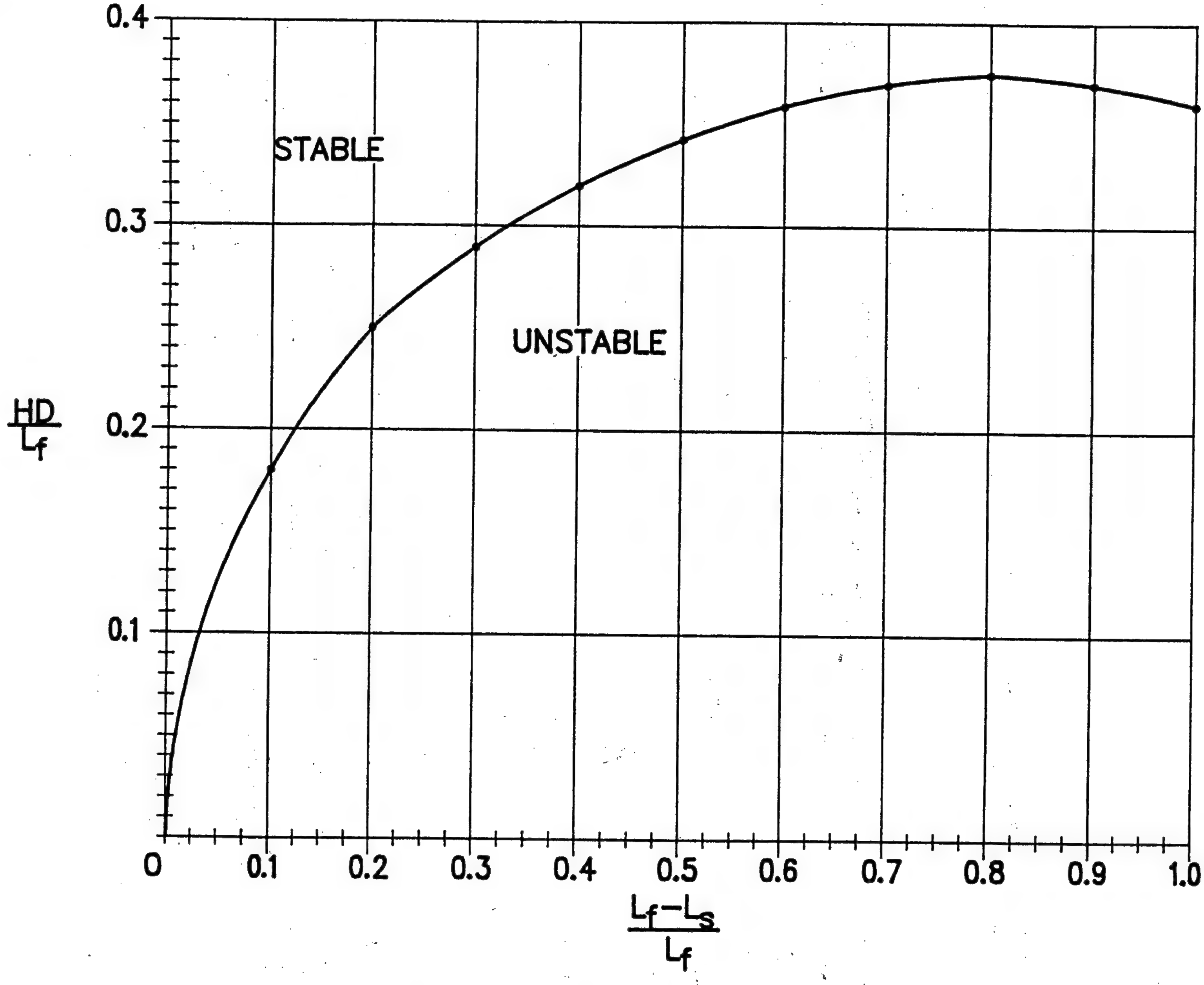


Figure 2. Critical values of HD/L_f to prevent buckling.

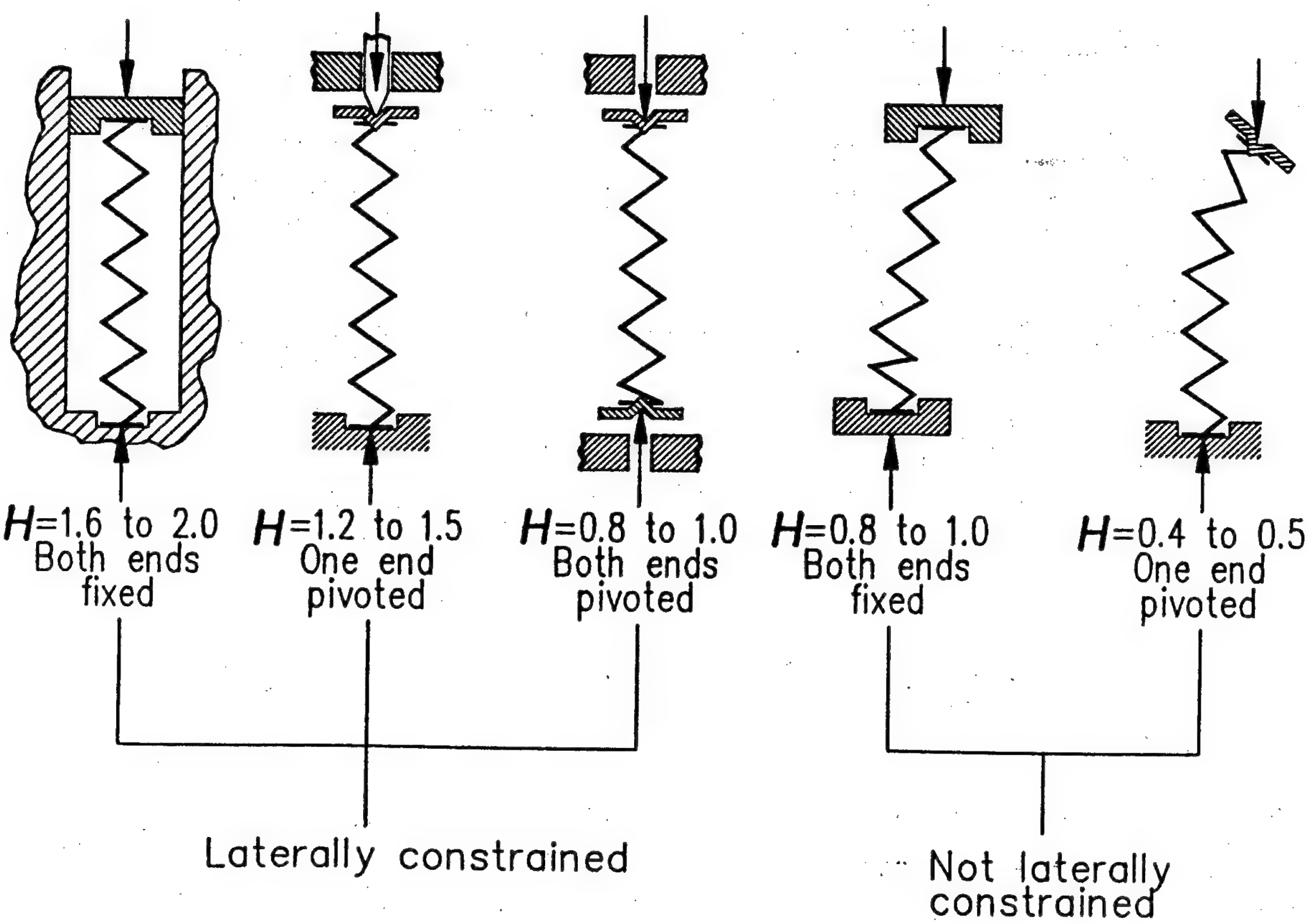
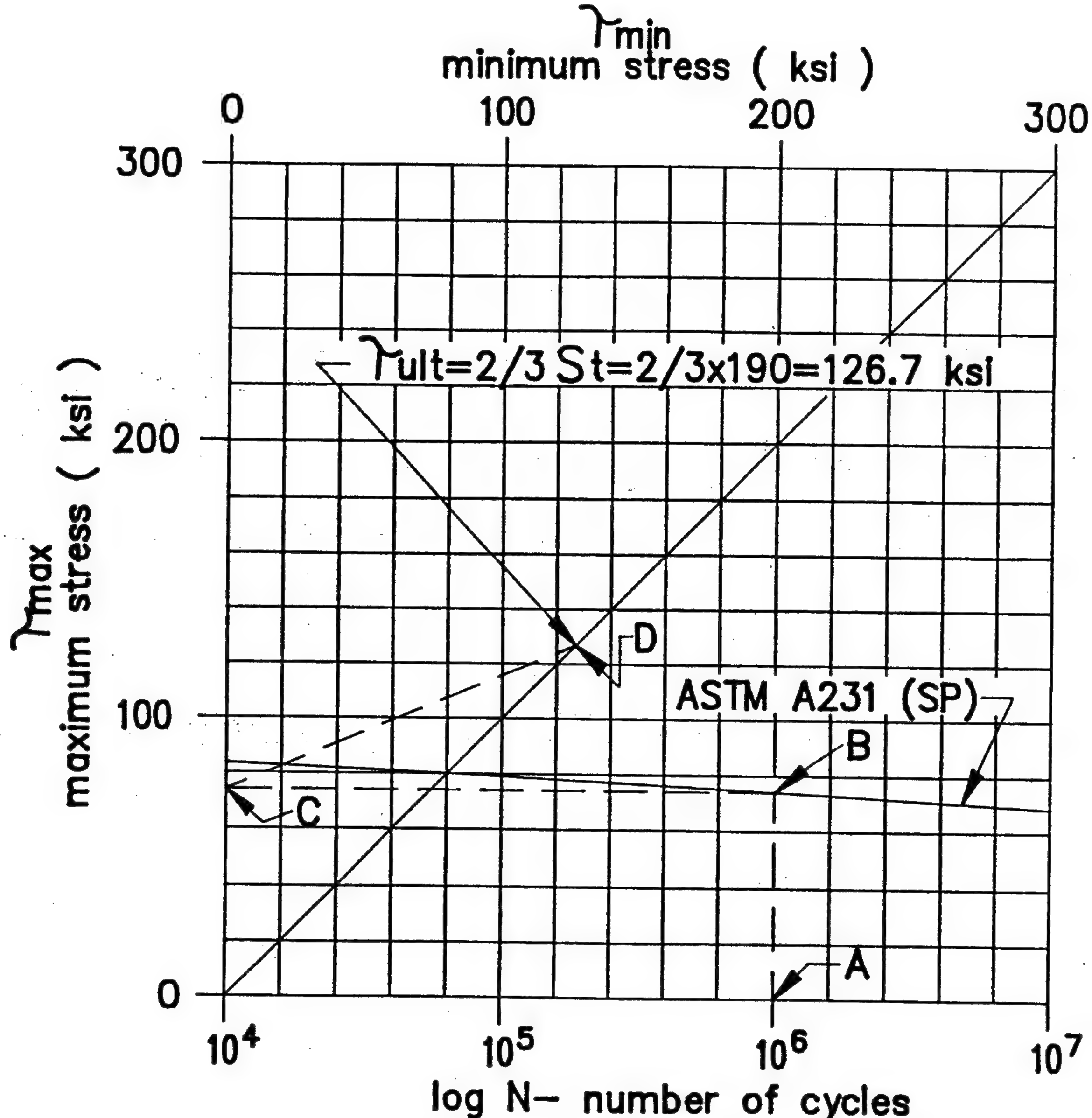


Figure 3. End fixation factor H for various end and constraint conditions.



τ_{max} vs N data from Table III for ASTM A231 material; shot peened.

cycles	τ_{max} (ksi)
10^5	79.8
10^6	74.1
10^7	68.4

Figure 4. Combined τ-N curve and modified Goodman diagram.

APPENDIX I

MAXIMUM SHEAR STRESS AT SOLID HEIGHT (τ_s) & CORRECTION FACTORS TO ACCOUNT FOR CURVATURE AND DIRECT SHEAR STRESS

I. MAX SHEAR STRESS:

Von Mises Distortion-Energy Yield Criterion ----------------Ref (3)
is $\tau_y = 0.577\ S_y$

To prevent yielding $\tau_s \leq \tau_y$

A. No Preset: $\tau_y = 0.577\ S_y$

(1) Cold Drawn Carbon Steel - $S_y \approx 0.8\ S_t$

$$\tau_s \leq 0.577\ S_y = 0.577 \times 0.8\ S_t$$

$$\tau_s \leq .45\ S_t$$

(2) Quenched & tempered Carbon & Alloy Steel - $S_y \approx 0.85\ S_t$

$$\tau_s \leq 0.50\ S_t$$

B. Preset: Residual stress increases effective strength by a factor of approximately 4/3 --------------------------------Ref (4)

$$\tau_y = 0.577 \times 4/3\ S_y = 0.77\ S_y$$

(1) Cold Drawn Carbon Steel $\tau_s \leq 0.61\ S_t$

(2) Quenched & Tempered Steel $\tau_s \leq 0.65\ S_t$

II. CORRECTION FACTORS

$$\tau = \frac{8\ P\ D}{\pi d^3} \times K_{w1\ or\ 2}$$

A. <u>No Preset or Fatigue Applications</u> - Use K_{w1}, which accounts for curvature and direct shear.

$$K_{w1} = \frac{4C-1}{4C-4} + \frac{0.615}{C} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Ref (1)}$$

B. <u>Preset</u> - Use K_{w2} which accounts for direct shear. Residual stress from preset is higher on the inside than the outside which offsets the higher applied stress on the inside due to curvature.

$$K_{w2} = 1 + \frac{0.5}{C} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Ref (1)}$$

APPENDIX II

Examples of Design Approaches

I. PRIMARY CONSIDERATIONS

A. Dimensional Stability - Spring has to be strong enough to withstand the maximum possible defection (free length minus solid height) without experiencing dimensional changes (plastic deformation). Analytically, this is expressed as follows: $\tau_s)_{max} \le \tau_y$.

B. Buckling - Is buckling allowed during the deflection?

C. Fatigue - Is fatigue life a concern?

II. EXAMPLES

Example 1. P + M = S (Buckling not Allowed)

1. Performance (P):

   a. Deflection between two loads (L_1 - L_2) equals 6".

   b. Load at L_1 is F_1 = 100 lbs.

   c. Buckling is not allowed. Both ends are fixed and spring is laterally contained - see Fig.2; H = 1.6.

   d. Ends are closed and ground.

   e. Fatigue life - 10^6 cycles minimum (shot peening will be specified).

2. Material (M): ASTM A231

3. Space (S): Determine d, OD, L_f, L_s. The size of the spring is determined by the performance and material requirements; however, selection of the deflection range will also affect the size. What portion of the total linear range will the spring operate?

$$\text{Let } y = \frac{L_f - L_1}{L_f - L_s} \quad \text{and} \quad x = \frac{L_f - L_2}{L_f - L_s}.$$

4. Calculations:

a. <u>Deflections & Spring Rate</u>

Let $x = .85$, $y = .15$:

$L_1 - L_2 = .7\,(L_f - L_s)$; $L_f - L_s = \underline{8.57"}$

$L_f - L_1 = \underline{1.285}$; $L_f - L_2 = \underline{7.285"}$

$$k = \frac{F_1}{L_f - L_1} = \underline{77.82 \text{ lb/in.}}$$

$F_2 = \underline{566.9\text{lb}}$

b. <u>Fatigue</u> - Calculate τ_2 and τ_1 from the modified Goodman Diagram - Ref. (1). Since the fatigue data available (τ_{max} v log N in Table III) is based on R = O, a Goodman diagram has to be combined with available fatigue data to determine the τ_2 and τ_1 combinations that would provide the desired life. A linear scale is used to plot minimum stress (abscissa) vs. maximum stress (ordinate) and a 45° line is drawn from the origin of the plot. The τ_{max} vs.log N curve is plotted from the data in Table III with the ordinate as τ_{max} and the abscissa representing log N. On the 45° line, point D corresponds

to the ultimate shear or torsional strength estimated to be two-thirds of the axial ultimate strength (Ref.1) . A vertical line is drawn from the desired life value (Point A) on the log N scale (10^6 cycles required) to the τ_{max} vs. log N curve (Point B). A horizontal line is drawn from Point B to intersect the ordinate (Point C) which is maximum stress (74.1 ksi) to provide 10^6 cycles when R = 0.

The line C-D represents the stress combinations (τ_1 and τ_2) allowed for 10^6 cycle life.

Equation for line C-D:

$$\tau_2 = \text{slope} \times \tau_1 + 74.1$$

$$\text{slope} = \frac{126.7 - 74.1}{126.7 - 0} = 0.415$$

$$\tau_2 = 0.415\,\tau_1 + 74.1$$

c. Relationship between d & D:

$$\tau_1 = \frac{F_1}{F_2} \times \tau_2$$

$$\tau_1 = .1764\,\tau_2$$

$$\tau_2 = 79.95 \text{ ksi}$$

$$\tau_2 = 8F_2\,DK_w / \pi d^3$$

$$DK_w / d^3 = 55.3823$$

$$C\,K_w = 55.3823 \times d^2$$

d. <u>Buckling:</u> $\dfrac{L_f - L_s}{L_f}$ vs. $\dfrac{HD}{L_f}$ ______________ per Fig 3.

Select preferred values of d from Table II, calculate C from equation derived in Step b, calculate n, L_s, L_f, $\dfrac{L_f - L_s}{L_f}$, $\dfrac{HD}{L_f}$.

Determine if selected value of d will result in a stable spring (see Fig. 3).

Select available wire diameter - let d = .406

Assume spring is not preset hence use K_{w1}: C. K_{w1} = 9.129

Trial and error solution : C = 7.65 ; D = 3.106

$$n: \quad n = \frac{Gd}{8\ kC^3} = \frac{11.5 \times 10^6 \times .406}{8 \times 77.82 \times 7.65^3} = 16.75$$

L_s: $L_s = (n + 2)\ d = 7.61$

L_f: $L_f = L_s + 8.57 = 16.18$

$\dfrac{L_f - L_s}{L_f} = .53$; $\dfrac{HD}{L_f} = .31$: From Fig 3, spring is not stable!

Increase d to next largest available size: Let d = .437

C : C = 9.12 ; D = 3.985 , OD = 4.422

n : n = 10.6

L_s : L_s = 5.506

L_f : L_f = 14.076

$\dfrac{L_f - L_s}{L_f} = .61$; $\dfrac{HD}{L_f} = .45$

Fig 3 - <u>Spring is Stable !</u>

e. Strength : $\tau_s = \dfrac{8\ F_s\ D\ K_{w1}}{\pi d^3}$

$F_s = k\ (L_f - L_s)$

F_s = 666.9 lb

$K_{w1} = 1.1597$

No preset : $\tau_s \leq .50\ S_T$

Min. S_T per ASTM A231 is 190 ksi ; $.5\ S_T$ = 95 ksi

τ_s = 94.04 ksi ; hence preset is not required

If the portion of the total linear range that the spring operates is smaller such as 20% to 80% (y=.2,x=.8) the total deflection would increase and the spring rate would decrease. Since stress ratio (τ_1/τ_2) would increase (100/400 vs 100/566.9), the stress values allowed to assure the fatigue life (10^6 cycles) would increase. For example if R=0 (τ_1= 0) then τ_2 would be 74.1 ksi whereas if R=1 ($\tau_1 = \tau_2$) then $\tau_1 = \tau_2$ = 126.7 ksi. An increase of stress values allows the spring index (C) to increase hence a larger mean coil diameter, less coils, a shorter free length and a more stable spring. A comparison of the spring dimensions (D, L_s, L_f) and stability characteristic for a specific wire diameter (d = .406) for the two deflection ranges (.15-.85 and .2-.8) is provided:

Deflection range	k	C_{max}	D	n	L_s	L_f	Stable
.15-.85	77.82	7.65	3.106	16.75	7.61	16.18	No
.2-.8	50	11.95	4.85	6.84	3.59	13.59	Yes

Example 2 P + M = S (Buckling Allowed and Fatigue life not a concern)

1) Performance (P):

a) $L_1 - L_2 = 6"$

b) $F_1 = 100$ lbs

c) Ends are closed and ground

2) Material (M) : ASTM A231

3) Space (S) : Determine d, OD, L_f, L_s.

4) Calculations:

a) <u>Deflection & Spring rate</u> (same as Example 1)

Let x = .85 , y = .15

$L_f - L_s = 8.57$

$L_f - L_1 = 1.285$

$L_f - L_2 = 7.285$

k = 77.82 lb/in

b) <u>Stress at solid height (τ_s)</u> : Spring is NOT preset

$$\tau_s = \frac{8\ F_s\ D\ K_{w1}}{\pi\ d^3}$$

$F_s = k\ (L_f - L_s) = 666.9$ lb

$\tau_s \leq .5\ S_t\ ;\ S_t \approx 190$ ksi

$\tau_s \leq 95$ ksi

Let $\tau_s = 90$ ksi

$$\frac{D\ K_{w1}}{d^3} = \frac{\pi\ \tau_s}{8\ F_s} = 52.996$$

$C\ K_{w1} = 52.996\ d^2$

Let $d = \underline{.437}$

$C K_{w1} = 10.1206$

$C = 8.66$

$D = 3.784$. $OD = 4.221$

$$n = \frac{Gd}{8kC^3} = 12.43$$

$L_s = 6.306$, $L_f = 14.876$

$L_1 = 13.591$, $L_2 = 7.591$

c) <u>Buckling</u> - yes or no? ; Fig 3

$$\frac{L_f - L_s}{L_f} = .57$$

= <u>Spring is stable</u>!

$$\frac{HD}{L_f} = .40$$

d) <u>Fatigue Life</u> ? Fig 4

$\tau_2 = \text{slope} \times \tau_1 + \tau_{max}$

$\tau_2 = 76.51$

$\tau_1 = 13.49$

Slope = .4433

τ_{max} at R=0 is <u>70.53</u>

Fatigue Life > 10^6 cycles

Summary of Examples 1 & 2.

	d	OD	k	n	L_f	τ_s	Life
Example 1	.437	4.422	77.82	10.6	14.076	94.04	10^6
Example 2	.437	4.221	77.82	12.4	14.876	90.0	$>10^6$

Example 3. $S + M = P$ (Buckling not Allowed)

1. Space (S) : a. Spring to work freely in 2" diameter bore.

   b. Free length - 9".

   c. Buckling is not allowed - Both ends are fixed and spring is laterally constrained (Fig 2, H = 1.6).

   d. Ends are closed and ground.

2. Material (M) : ASTM A231.

3. Performance (P): a. Determine loads, compressed lengths, and if preset is required.

   b. Determine fatigue life (shot peen vs. not shot peened).

4. Calculations:

   a. D & d values to accommodate 2" bore:

$$OD_{spring} = OD_{bore} - .05\ OD_{bore}$$

(approximately 5% expansion to prevent binding in the 2" bore) ------------ Ref (1)

$$= \underline{1.9}$$

Select available wire diameter from Table II; calculate D & C:

$$D = 1.9 - d$$

d	D	C	
.375	1.525	4.066	- C should be 4.0 minimum hence .375 is the largest diameter selected.
.281	1.619	5.76	
.207	1.693	8.18	
.162	1.738	10.73	

b. Buckling : Calculate HD / L_f = for each value of d and from Fig 3 determine the maximum value of $\dfrac{L_f - L_s}{L_f}$ to prevent buckling. Also calculate L_s, n, and k.

e.g.: select d = .375

$$\frac{HD}{L_f} = \frac{1.6 \times 1.525}{9} = .27$$

From Fig 3 : $\dfrac{L_f - L_s}{L_f.} = .25$ max.

L_s: $L_s = L_f - .25\ L_f$

L_s = <u>6.75</u>

n: $(n + 2)\ d = L_s$

n = <u>16</u>

Increase n ≈ .5 coils to provide safety from buckling.

Let n = <u>16.5 coils</u>

New $L_s = (16.5 + 2) \times .375 = 6.9375$

k: $k = \dfrac{Gd}{8\ nC^3}$ = <u>486 lb/in</u>

d	HD/L_f	Max $(L_f-L_s)/L_f$	n Calc	n selected	k
.375	.27	.25	16.0	16.5	486.0
.281	.29	.30	20.4	21.0	100.6
.207	.30	.34	26.7	27.0	20.1
.162	.31	.35	34.1	34.5	5.5

c. Loads, Compressed Lengths, and Stress at solid height:

Select the deflection range the spring will operate!

Let $L_f - L_1 = .2 \ (L_f - L_s)$ and $L_f - L_2 = .8 \ (L_f - L_s)$

i.e. 20% and 80% of the total deflection which is within the linear or elastic zone.

e.g. select d = .375

$L_f - L_1 = .2 \ (L_f - L_s) = .41$

$L_1 = \underline{8.59}$

$L_f - L_2 = 1.65$

$L_2 = 7.35$

$F_1 = k \ (L_f - L_1) = \underline{199.3 \text{ lbs}}$

$F_2 = k \ (L_f - L_2) = \underline{801.9} \text{ lbs}$

$F_s = k \ (L_f - L_s) = \underline{1002.1} \text{ lbs.}$

$$\tau_s = \frac{8 \times 1002.1 \times 1.525 \times 1.396}{\pi \times .375^3}$$

$\tau_s = \underline{103}$ ksi

d	L_1	L_2	F_1	F_2	F_s	τ_s	$.5S_T$	Preset Required
.375	8.59	7.35	199.3	801.9	1002.1	103	100	yes
.281	8.49	6.97	51.3	204.2	255.2	60	102.5	no
.207	8.4	6.6	32.2	48.2	60.2	35.6	105	no
.162	8.38	6.53	3.4	13.5	16.9	19.9	112.5	no

d. Fatigue Life: For each value of d, calculate τ_1 and τ_2 and plot fatigue data (Table III) on Fig. 4. Draw a line from point D (torsional strength) through (τ_1, τ_2) to intersect the ordinate. Draw a horizontal line from the ordinate to intersect the τ_{max} vs. log N curves for the ASTM A231 material. Draw vertical lines from the τ_{max} vs. log N intersect points to the abscissa (log N) to determine the fatigue lives.

e.g. d = .375 F_1 = 199.3 ; τ_1 = 20.5 ksi

F_2 = 801.9 ; τ_2 = 82.4 ksi

Figure 4 - N > 10^6 cycles

As d decreases, τ_1 and τ_2 decrease, hence N increases.

CONCLUSION: The space and no buckling limitations result in specific values of d, D, n and k. The selection of where the spring will operate in the deflection range, further defines the specific load/compressed length values.

Example 4. S + M = P (Same space and material as Example 2 except buckling is allowed)

If buckling is allowed, the approach is to calculate the force at solid height (F_s) that correlates with the maximum stress allowed at solid height. Then the number of active coils would be calculated from the equation:

$$F_s = k\ (L_f - L_s) = \frac{Gd^4}{8nD^3}\ [\,9 - (n + 2)d\,] .$$

The spring rate would then be determined and F_1, L_1, F_2, L_2 would be dependent upon selection of the operating deflection range.

1. Calculations: Select available wire diameter from Table II; calculate D.

e.g. Let $d = .281$, $D = 1.619$ as previously calculated for the space limitation of 2" bore diameter (See Example 3).

a. Max Force - Assume no preset required

τ_s max $\leq = .5S_T$; $S_T = 205$Ksi for $d = .281$ (ASTM A231)

Let τ_s max $= .45\ S_t$ to provide safety from permanent deformation

$$F_s = \frac{\tau_s\ \pi d^3}{8 \times D \times K_{w1}} = \frac{.45 \times 205 \times \pi \times .281^3}{8 \times 1.619 \times 1.2643}$$

$F_s = \underline{392.4 \text{ lbs}}$

b. Number of Coils (n)

$$F_s = k\ (L_f - L_s) = \frac{Gd^4}{8nD^3}\ [\,9 - (n+2)d\,]$$

$$392.4 = \frac{11.5 \times 10^6 \times .281^4}{8 \times 1.619^3 \times n}\ [\,9 - (n + 2) \times .281\,]$$

$n = 18.1$

c. Spring rate (k)

$$k = \frac{Gd^4}{8nD^3}$$

$$k = \frac{11.5 \times 10^6 \times .281^4}{8 \times 18.1 \times 1.619^3}$$

$k = \underline{\underline{116.7}}$ lb/in.

d. F_1, L_1, F_2, L_2

Let deflection range be 20 - 80% of total

Let $L_f - L_1 = .2\ (L_f - L_s)$; $L_s = (n+2)d = 5.648$

$L_1 = \underline{8.33"}$; $F_1 = \underline{78.2 \text{ lb.}}$

Let $L_f - L_2 = .8\ (L_f - L_s)$

$L_2 = \underline{6.32}$; $F_2 = \underline{312.75 \text{ lb.}}$

e. <u>Fatigue Life:</u> See Example 3, Step 4.d

$$\tau_1 = \frac{8\ F_1 D\ K_{w1}}{\pi d^3} = \frac{8 \times 78.2 \times 1.619 \times 1.2643}{\pi \times .281^3} = \underline{18.37} \text{ ksi}$$

$\tau_2 = \underline{73.48}$ ksi

From Fig. 4 - $\underline{\underline{N > 10^6 \text{ cycles}}}$

CONCLUSION: Space limitations provide specific values of D & d. The maximum solid stress limitation provides the number of coils and a spring rate. The choice of deflection range further defines load/compressed length values.

Comparison of Example 3 & 4 for d = .281

	<u>n</u>	<u>k</u>	<u>L_1</u>	<u>L_2</u>	<u>F_1</u>	<u>F_2</u>	<u>τ_s</u>
Buckling Not Allowed (Ex. 3)	21.0	100.6	8.49	6.97	51.3	204.2	60
Buckling Allowed (Ex. 4)	18.1	116.7	8.33	6.32	78.2	312.75	92.25

Example 5. $P + S = M$ (Buckling allowed)

1. Performance (P): (same as example 1 except buckling is allowed)

   a. Deflection between two loads: $L_1 - L_2 = 6"$

   b. Load at L_1 : $F_1 = 100$ lbs

   c. Buckling is allowed.

   d. Ends are closed and ground

2. Space (S):

   a. Spring to work freely in 2" diameter bore

   b. Free length - 10" max

3. Material (M): Select material capable of withstanding applied torsional stress when spring is compressed to solid height (L_s).

4. Calculations:

   a. Deflections, L_1, L_2, L_s, and spring rate:

$$L_1 - L_2 = 6 = x\ (L_f - L_s)$$

$$L_s = 10 - 6/x$$

Since $L_s > 0$, $x > .6$

Max value of x for linearity of load vs. deflection is .7.- See Example 1.

$$L_f - L_s = \frac{6}{.7} = \underline{8.57}$$

$L_s = \underline{1.43}$

Let $L_f - L_1 = .15\ (L_f - L_s) = 1.285$; $L_1 = \underline{8.715}$

and $L_f - L_2 = .85\ (L_f - L_s) = 7.285$; $L_2 = \underline{2.715}$

$k = F_1 / L_f - L_1 = \underline{77.82}$ lb/in

b. Space : determine maximum sizes (D & d) to have

$OD_{spring} = 2 - .05 \times 2 = \underline{1.9}$

$k = Gd/ 8nC^3$ or $d/n = 8kC^3 / G = 54.135 \times 10^{-6} C^3$

$d = L_s / n + 2$

$$\frac{1.43}{n\ (n + 2)} = 54.135 \times 10^{-6} C^3$$

<u>Trial & Error</u> - Calc maximum C such that D+d ≈ 1.9

Let C = 8.76 ; n = 5.35

d = 1.43 / 5.35 + 2 = .195

D = C x d = 1.708

D + d = <u>1.903</u> ; close enough !

NOTE: C < 8.76 would accommodate the space allowed, however the lower the value of C, the smaller the value of d to maintain k hence the higher the value of τ_s.

c. Maximum tensional stress : $\tau_s = 8F_s\ D\ K_{w2} / \pi d^3$

$F_s = k\ (L_f - L_s) = 77.82\ (10 - 1.43) = \underline{666.9}$

$\tau_s = \underline{413.5}$ ksi

CONCLUSION: Since $\tau_s \leq .65\ S_t$ for a preset, Q & T, alloy steel spring to avoid plastic deformation

$S_t \geq 413.5 /.65 = 636$ ksi. Material does <u>NOT</u> exist !

Example 6: P + S = M (Buckling Not Allowed) and allow larger Space than Example 5.

1. Performance: As specified in Example 6: $L_1 - L_2 = 6"$, $F_1 = 100$ lbs, Buckling not allowed, H = 1.6, closed and ground ends.

2. Space: a. Spring to work freely in 5" diameter bore.

   b. Free length - 15" max.

3. Material: Select material to withstand $\tau_s \le .65\ S_t$ (preset)

4. Calculations:

   a. Establish percent of total deflection that $L_1 - L_2$ represents:

$$L_1 - L_2 = x\ (L_f - L_s) = 6$$

$$L_s = 15 - \frac{6}{x}$$

Since $L_s > 0$, $x > .4$

Let x = .7 which is the maximum for linearity between load and deflection

$L_f - L_s = \underline{8.57}$; $L_s = \underline{6.43}$

Let $L_f - L_1 = .15\ (L_f - L_s) = \underline{1.285}$, $L_1 = \underline{13.715}$

Let $L_f - L_2 = .85\ (L_f - L_s) = \underline{7.285}$, $L_2 = \underline{7.715}$

   b. Determine k:

$$k = \frac{F_1}{L_f - L_1} = \underline{77.82}\ \text{lb/in}$$

c. Determine C that will fit into the allowed space:

$OD_{spring} = .95 \times OD_{bore} = .95 \times 5 = \underline{4.75}$

$$\frac{d}{n} = \frac{8kC^3}{G} = 54.135 \times 10^{-6}C^3$$

$d = Ls / (n + 2)$

<u>Trial & Error</u>

Calculate maximum C such that $D + d \leq 4.75$

Let d = .500 : n = 10.86, C = 9.472, D = 4.736, OD = 5.236 (Too Large)

Let d = .469 : n = 11.7, C = 9.042, D = 4.241, OD = 4.71 (Close enough)

d. Determine τ_s:

$$\tau_s = \frac{8\ F_s\ D\ K_{w2}}{\pi d^3} \ : \ K_{w2} \text{ used since spring will be preset}$$

$$\tau_s = 8 \times \frac{666.9 \times 4.241 \times 1.055}{\pi \times .469^3}$$

$\tau_s = \underline{73,673 \text{ psi}}$

S_t has to be $\geq \dfrac{\tau_s}{.65} = 113,343$ psi - All available material has $S_t > 113$ ksi

e. Determine if C = 9.042 will buckle (Fig. 3, H = 1.6)

<u>C</u>	<u>n</u>	<u>d</u>	<u>D</u>	$\frac{L_f - L_s}{L_f}$	<u>HD/L_f</u>	<u>Stable?</u>
9.042	11.7	.469	4.241	.57	.45	YES i.e. Spring will not buckle!

f. Minimum C to prevent buckling:

$$\text{Figure 3:} \quad \frac{L_f - L_s}{L_f} = \frac{8.57}{15} = .57 \;; \quad \frac{HD}{L_f} \geq .36 \text{ to prevent buckling}$$

$$D \geq \frac{.36 \times 15}{1.6} = \underline{3.375} \text{ min}$$

$$\frac{d}{n} = 54.135 \times 10^{-6} \, C^3 \text{ -------------See Step c,}$$

$$= 54.135 \times 10^{-6} \left(\frac{D}{d}\right)^3$$

$$d^4 = \underline{2081.13 \times 10^{-6} \, n}$$

$$\text{Also} \quad (n+2)d = L_s \;; \quad n = \frac{6.43}{d} - 2$$

$$\therefore \; d^4 = \frac{13381.67 \times 10^{-6}}{d} - 4162.26 \times 10^{-6}$$

Trial & Error - d = .412

$$\therefore \; C_{min} = \frac{D}{d} = \frac{3.375}{.412} = \underline{8.19} \;; \quad (OD = 3.787)$$

$$\tau_s = \underline{86,960} \text{ psi}$$

S_t has to be $\geq \dfrac{\tau_s}{.65}$ = 133,784 psi - all available material has S_t > 134 ksi

CONCLUSION: The spring index range is 8.19 to prevent buckling and 9.042 to fit into the allowed space. The load/compressed length values are based on the selection of where the spring will operate in the deflection range. The stress at solid height is less than the allowable level (preset or not preset), hence the choice for material selection is unlimited.

APPENDIX III

SPECIFICATION REQUIREMENTS & TOLERANCES

I. REQUIREMENTS:

A. Material	-	ASTM Spec No. (See Table I).
B. Wire Diameter (d)	-	tolerance is controlled in the ASTM material spec. Stress (dimensional stability and fatigue life) is dramatically affected by d ($\tau \approx 1/d^3$)
C. Coil Diameter (D)	-	tolerance is per Table 5-5, (Ref 2) ($\tau \approx D$), Coil diameter directly affects stress, spring rate, ($k \approx 1/D^3$) and buckling.
D. Two loads at compressed lengths	-	Necessary for controlling performance. Effectively the spring rate is controlled! Tolerance for the loads compensate for the variation in k due to tolerances for d, OD and in L_f per Table 5-4 (Ref 2). The tolerance for L_f is to allow for variation in either pitch or number of coils. Examples of load tolerance & calculations are in Section III of this Appendix III
E. Type of Ends	-	The type of ends affects L_s hence the total deflection range and affects buckling.
F. Preset, Shot Peening and Protective Coating	-	Preset may be required for elastic stability, shot peening improves high cycle fatigue life, protective coatings (see MIL-S-13572) improve corrosion resistance especially for springs exposed to the atmosphere - see Ref 4).

II. REFERENCE VALUES

A. Free length & total number of coils	-	Should be Reference values to allow for manufacturing variations and inaccuracies of the theoretical equations.

III. ESTABLISH LOAD TOLERANCES:

A. P + M = S (Buckling Not Allowed): (see Example 1 of Appendix II).

(1) Performance (P):

(a) $L_1 - L_2 = 6"$

(b) $F_1 = 100$ lb.

(c) Buckling NOT allowed. Ends fixed, H = 1.6

(d) Ends are closed and ground.

(e) Fatigue life - 10^6 cycles minimum (shot peening specified).

(2) Material (M) : ASTM A231.

(3) Space (S) : deflection range of 15-85% of total.

(a) $L_f - L_s = 8.57$, $L_f - L_1 = 1.285$, $L_f - L_2 = 7.285$

(b) k = 77.82 lb/in.

(c) d = .437", C = 9.15, D = 3.998 OD = 4.435

(d) n = 10.5

(e) $L_f = (14.03)$, $L_s = (5.46)$

(4) Tolerances: d, OD, L_f

(a) d : ASTM A231 specifies ± 0.002

d = 0.437 ± 0.002

(b) OD : Table 5-5 (Ref 1) specifies ± 0.042

OD = 4.435 ± 0.042

(c) L_f: Table 5-4 (Ref 1) specifies $\pm 0.028 \times L_f = \pm 0.39$
$L_f = 14.03 \pm .39$

(d) F_1 & F_2 : Load tolerance is controlled by the variation in k due to tolerances for d, OD and L_f.

(5) Calculations: k & F

$$k_{min}: \quad k_{min} = \frac{Gd^4_{min}}{8nD_{max}^3}$$

$$D_{max} = OD_{max} - d_{min}$$

$$= 4.477 - .435$$

$$D_{max} = \underline{4.042}$$

$$k_{min} = \underline{74.23 \text{ lb/in.}}$$

$$k_{max}: \quad k_{max} = \frac{Gd^4_{max}}{8nD_{min}^3}$$

$$D_{min} = OD_{min} - dmax$$

$$D_{min} = \underline{3.954}$$

$$k_{max} = \underline{83.13 \text{ lb/in}}$$

$$F_{1\ min} = k_{min} (L_{fmin} - L_1) \ ; \quad L_1 = \underline{12.745}$$

$$= \underline{66.43 \text{ lb}}$$

$$F_{1\ max} = \underline{139.24 \text{ lb}}$$

$$F_1 = \underline{103 \pm 36 \text{ lb}}$$

$$F_2 = \underline{532 \pm 60 \text{ lb}} \ ; \quad L_2 = \underline{7.285}$$

(6) List of Requirements:

(a) Spring per MIL-S-13572 Type I, Grade __*__.

(b) Material - wire per ASTM A231 ; $d = 0.437 \pm 0.002$

(c) Coil diameter - 4.435 ± 0.042

(d) Loads at compressed lengths: $F_1 = 103 \pm 36$ lb @ $L_1 = 12.745$

$F_2 = 532 \pm 60$ lb @ $L_2 = 7.285$

(e) Ends - closed and ground

(f) Shot peen per MIL-S-13165 (steel shot)

(g) Preset required

(h) Free length - (14.03)

(i) Total number of coils - (12.5)

* Choice of A or B

B. S + M = P (Buckling Allowed): (see Example 3 of Appendix II)

(1) Space (S):

(a) OD = 1.9

(b) $L_f = 9$

(c) d = .281, D = 1.619, n =18.1

(d) $L_1 = 8.33$, $L_2 = 6.32$

(e) Ends closed and ground.

(2) Material (M): ASTM A231

(3) Tolerances :

(a) d : ASTM specifies ± .002

$$d = .281 \pm .002$$

(b) OD : Table 5-5 (Ref 1) specifies ± .015

OD = 1.900 ± 0.015

(c) L_f: Table 5-4 (Ref 1) specifies ± .025 L_f = ± .225

Lf = 9 ± .225

(d) F_1 & F_2 : Load tolerance is controlled by the variation in k due to tolerance for d & OD and L_f.

(4) Calculation: k & F

$$k : k_{min} = \frac{Gd^4_{min}}{8nD^3_{max}} \; ; \; k_{max} = \frac{Gd^4_{max}}{8nD^3_{min}}$$

$D_{max} = OD_{max} - d_{min} = \underline{1.636}$

$D_{min} = OD_{min} - d_{max} = 1.602$

$k_{min} = \underline{109.9}$

$k_{max} = \underline{123.9}$

F_1 : $F_{1\,min} = k_{min} (L_{f\,min} - L_1) = \underline{48.9}$

$F_{1\,max} = \underline{110.9}$

$F_1 = \underline{80 \pm 31}$ at $L_1 = 8.33$

F_2 : $F_{2\,min} = \underline{163.8}$

$F_{2\,max} = \underline{240.4}$

$F_2 = \underline{202 \pm 38}$ at $L_2 = \underline{6.32}$

(5) List of Requirements

(a) Spring per MIL-S-13572 Type I, Grade ____

(b) Material - wire per ASTM A231, d = .281 ± .002

(c) Coil diameter - 1.900 ± .015

(d) Loads at compressed lengths: $F_1 = 80 \pm 31$ @ $L_1 = 8.33$

$F_2 = 202 \pm 38$ @ $L_2 = 6.32$

(e) Ends - closed and ground

(f) Free length - (9)

(g) Total number of coils - (20.1)

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